IMPACT: International Journal of Research in Engineering & Technology (IMPACT: IJRET)
ISSN(E): 2321-8843; ISSN(P): 2347-4599
Vol. 4, Issue 2, Feb 2016, 1-6

THE METHOD OF FICTITIOUS AREAS FOR MODELING OF HYPERBOLIC EQUATIONS

TUKENOVA LYAILYA MURATBEKOVNA & SKAKOVA AIGUL JUMAKHANOVNA
Research Scholar, Turar Ryskulov New Economic University, Almaty, Kazakhstan

ABSTRACT

In this article we examine the method of fictitious areas for the non-liner hyperbolic equations. The estimation of rate of convercence decisions is recaived. In some cases the unimproved estimation of convergence rate of the decision is received.

KEYWORDS: Hyperbolic Equation, The Method of Fictitious Areas, Filtration, Boundary Value, Unimproved Estimation

Mathematics Subject Classification Computational Mathematics

INTRODUCTION

Today the method of fictitious areas is widely used for the numerical simulation of problems in mathematical physics in areas of complex geometry. For the equation of Navier-Stokes with heterogeneous limited conditions the monograph [1] is devoted, for models of filtration- the monograph [2], for the problems of mathematical physics- [3].

The method of fictitious areas for linear equations of mathematical physics, excluding hyperbolic equations, is well explored (for example, monograph [2] and the sources mentioned there). In non-stationary problems unimproved estimations of speed in most cases are not received. Moreover, for non-linear limited problems the technology of obtaining unimproved estimations of convergence speed according to the known methods is not available. But there are not enough scientific materials devoted to the method of fictitious areas for boundary value problems of hyperbolic equations. In this monograph the justification of the method of fictitious areas for initial limited problems of non-linear hyperbolic equations is given and the speed estimation of solving the auxiliary equation is obtained. In some cases the speed of convergence unimproved in defined order. The new way of improving the speed estimation in the norm L_2 is offered in this monograph.

PROBLEM FORMULATION

Let examine the problem of Dirikhle in area of $\Omega\subset R^n$c with the limit S for non-linear differential equations like

$$\frac{\partial^2 v}{\partial t^2} = \Delta v - |v|^2 + f(t,x) \tag{1}$$

$$v|_{t=0} = v_0(x) \quad v_{t|t=0} = v_1(x), \tag{2}$$

$$v_{|S} = 0\ . \tag{3}$$

The theorem of existence of generalized solution and the diffential properties of the equations are well explored in the monograph [4].

According to the method of fictitious areas the equation of the problem by using the smallest coefficient in auxiliary area D=$\Omega \cup D_0$ in the limit$S_1, S_1 \cap S = \emptyset$will be this

$$\frac{\partial^2 \upsilon^\varepsilon}{\partial t^2} = \Delta \upsilon^\varepsilon - |\upsilon^\varepsilon|^p \upsilon^\varepsilon - \xi(x)\left(\frac{\gamma}{\varepsilon^\alpha}\upsilon^\varepsilon + \frac{1-\gamma}{\varepsilon^\beta}\upsilon_t^\varepsilon\right) + f(t,x) \,, \tag{4}$$

$$\upsilon^\varepsilon|_{t=0} = \upsilon_0(x),$$

$$\upsilon^\varepsilon|_{t=0} = \upsilon_1(x), \tag{5}$$

$$\upsilon^\varepsilon|S_1 = 0, \tag{6}$$

Where

$$\xi(x) = \begin{cases} 0, & x \in \Omega, \quad \alpha,\beta \geq 0, \quad 1 \geq \gamma \geq 0 \\ & 1, \quad x \in D_0 \end{cases}$$

$\upsilon_0(x),\ \upsilon_1(x),\ f(t,x)$ - *The zeros out of Ω.*

As a result, the following theorem can be defined.

Theorem 1

Let’s take $\rho < \frac{2}{n-2}, \upsilon_0(x) \in_{W_2^2}^0 (\Omega), \upsilon_1(x) \in_{W_2^1}^0 (\Omega), f_t \in L_2(0,T;L_2(\Omega))$ then the equation (7) takes place in solving problem

$$\|\upsilon_{tt}^\varepsilon\|_{L_\infty(0,T;L_2(D))} + \|\upsilon_x^\varepsilon\|_{L_\infty(0,T;L_\infty(D))} + \frac{\gamma}{\varepsilon^\beta}\|\upsilon_{tt}^\varepsilon\|^2_{L_2(0,T;L_2(D_0))} +$$

$$+ \frac{(1-\gamma)}{\varepsilon^\alpha}\|\upsilon_t^\varepsilon\|^2_{L_\infty(0,T;L_2(D_0))} \leq C < \infty \,. \tag{7}$$

Proof: Let (3) be multiplied by υ_t^ε and integrated in the area D, then we have the estimation

$$\|\upsilon_t^\varepsilon\|_{L_\infty(0,T;L_2(D))} + \|\upsilon_x^\varepsilon\|_{L_\infty(0,T;L_2(D))} + \frac{1}{p+2}\|\upsilon^\varepsilon\|_{L_\infty(0,T;L_{p+2}(D))}$$

$$+ \frac{\gamma}{\varepsilon^\beta}\|\upsilon_t^\varepsilon\|^2_{L_\infty(0,T;L_2(D_0))} + \tag{8}$$

By differentiating (4) in ***t*,** multiplying by $\upsilon_{tt}^\varepsilon$ and integrating in the area D we obtain

$$\frac{1}{2}\frac{d}{dt}\left(\|v_{tt}^{\varepsilon}\|^2+\|v_{xt}\|^2+\frac{(\gamma-1)}{\varepsilon^{\alpha}}\|v_t^{\varepsilon}\|^2\right)+\frac{\gamma}{\varepsilon^{\beta}}\|v_{tt}^{\varepsilon}\|^2+(p+1)(|v^{\varepsilon}(t)|^p v_t^{\varepsilon}(t),v_{tt}^{\varepsilon})=(f^{'}(t),v_{tt}^{\varepsilon}).$$

By using the inequality of Gelder we have

$$|(v^{\varepsilon}(t)^p v_t^{\varepsilon}(t),v_{tt}^{\varepsilon}))|\leq \||v^{\varepsilon}(t)|^p\|L_n(D)\|v_t^{\varepsilon}\|L_q(D)\|v_{tt}^{\varepsilon}\|L_2(D),$$

Where (as in the inflow theorem of Sobolev)

$$\frac{1}{n}+\frac{1}{q}+\frac{1}{2}=1$$

According to the condition $pn\leq q$, and from (8) we have

$$\||v^{\varepsilon}(t)|^p\|L_n(D)\leq \|v_x^{\varepsilon}\|_{L_{2(D)}}^p\leq C<\infty,$$

$$|(f^{'}(t),v_{tt}^{\varepsilon})_D|\leq \|f^{'}(t)\|L_2(D)\|v_{tt}^{\varepsilon}\|L_{2(D)}.$$

As a result, we obtain

$$\|v_{tt}^{\varepsilon}\|_{L_\infty(0,T;L_2(D))}+\|v_{tx}^{\varepsilon}\|_{L_\infty(0,T;L_2(\Omega))}+\frac{(\gamma-1)}{\varepsilon^{\alpha}}\|v_t^{\varepsilon}\|_{L_\infty(0,T;L_2(D_0))}++\frac{\gamma}{\varepsilon^{\beta}}\|v_{tt}^{\varepsilon}\|_{L_2(0,T;L_2(D_0))}\leq C<\infty \tag{9}$$

(9)The following theorem is appeared.

Theorem 2

Let all conditions of the first theorem be used. Then the only one solution of the problem (4)-(6), which has the property (9), is existing. It goes to the solution of the problem (1)-(3) in $\varepsilon\to 0$.

The theorem of solution existence is proved by the Galerkin's method and with the helpp of methods offered in monograph [4], the convergence of solution depends on the estimation (9). Then lets research the speed of convergence in the solution of problem (4)-(6). The following theorem is obtained

Theorem 3

Lets $f_t\in L_2(0,T;L_2(\Omega)), v_0(x)\in \dot{W}_2^2(\Omega), v_1(x)\in \dot{W}_2^1(\Omega)$, f(t,x), $v_0(x), v_1(x)$ –the continuous zero out of Ω, $\rho\leq\frac{n}{n-2}$.

Then this estimation takes place

$$\|v_t^{\varepsilon}-v_t\|_{L_\infty(0,T;L_2(\Omega))}^2+\|v^{\varepsilon}-v\|_{L_\infty(0,T;L_2(\Omega))}^2+\frac{\gamma}{\varepsilon^{\alpha}}\|v^{\varepsilon}-v\|_{L_\infty(0,T;L_2(D_0))}^2++\frac{(1-\gamma)}{\varepsilon^{\beta}}\|v_t^{\varepsilon}-v_t\|_{L_{2(0,T;L_2(D_0))}}^2\leq C\varepsilon^{x},$$

$$\|v^{\varepsilon}-v\|_{L_\infty(0,T;L_2(S))}\leq C\varepsilon^{x}.$$

$$If \int_0^t \left\| \frac{\partial \upsilon_{tt}}{\partial n} \right\|_{L_2(S)}^2 dt \le C < \infty,$$

Then

$$\|\upsilon_{tt}^{\varepsilon} - \upsilon_{tt}\|_{L_{\infty}(0,T;L_2(D))}^2 + \|\upsilon_{tx}^{\varepsilon} - \upsilon_{tx}\|_{L_{\infty}(0,T;L_2(D))}^2 + \frac{\gamma}{\varepsilon^{\alpha}} \|\upsilon_t^{\varepsilon} - \upsilon_t\|_{L_{\infty}(0,T;L_2(D_0))}^2 + + \frac{(1-\gamma)}{\varepsilon^{\beta}} \|\upsilon_{tt}^{\varepsilon} - \upsilon_{tt}\|_{L_2(0,T;L_2(D_0))}^2 \le \varepsilon^{x},$$

$$\|\upsilon_t^{\varepsilon} - \upsilon_t\| L_{\infty(0,T;L_2(S))} \le C \varepsilon^{x},$$

where

$$x = \begin{cases} \frac{\beta}{2}, if\ \gamma = 0, \\ \max\left\{\frac{\alpha}{2}, \frac{\beta}{2}\right\}, if\ \beta > 0, \alpha \ge 0, \beta \ge 0, \alpha + \beta > 0, 0 < \gamma < 1, \\ \frac{\alpha}{2}, if\ \gamma = 1, \alpha > 0. \end{cases}$$

By multiprlying (1) by function $\varphi(x) \epsilon \dot{W}_2^1(D)$ and integrating in the area ***Ω***, $\upsilon(x,t)$ −the continuous zero out of Ω. In a result we will obtain integral concordance

$$\left(\frac{\partial^2 \upsilon}{\partial t^2}, \varphi\right)_D + (\upsilon_x, \varphi_x)_D + (|\upsilon|^{\rho} \upsilon, \varphi)_D - \int_S \frac{\partial \upsilon}{\partial n} \varphi dS = (f(t,x), \varphi)_D. \quad (10)$$

Lets determine $\upsilon^{\varepsilon} - \upsilon = \omega\ and\ \varphi = \omega_t$. Then the (4)-(6) and (10) for ω will make the integral concordance

$$\frac{1}{2}\frac{d}{dt}\left(\|\omega_t\|_D^2 + \|\omega_x\|_D^2 + \frac{\gamma}{\varepsilon}\|\omega\|_{D_0}^2\right) + \frac{(1-\gamma)}{\varepsilon}\|\omega_t\|_{D_0}^2 +$$

$$+ (|\upsilon^{\varepsilon}|^{\rho}\upsilon^{\varepsilon} - |\upsilon|^{\rho}\upsilon, \omega_t)_D = \int_S \frac{\partial \upsilon}{\partial n} \omega_t dS. \quad (11)$$

Lets transform the additives on the right side (11), then integrate in t

$$\int_S \frac{\partial \upsilon}{\partial n} \omega_t ds = \int_S \frac{\partial \upsilon}{\partial n} \omega ds - \int_0^t \int_S \frac{\partial \upsilon_t}{\partial n} \omega ds. \quad (12)$$

By estimating (12) in inequality of inflow [5], we take

$$\left|\int_S \frac{\partial \upsilon}{\partial n} \omega ds\right| \le \|\omega\|_{L_2(S)} \left\|\frac{\partial \upsilon}{\partial n}\right\|_{L_2(S)} \le C \|\omega_x\|_{L_2(D_0)}^{\alpha_0} \|\omega\|_{L_2(D_0)}^{1-\alpha_0} \le C\sqrt{\varepsilon^{\frac{\alpha}{2}} \|\omega_x\|_{D_0}^2 + \varepsilon^{\frac{-\alpha}{2}} \|\omega\|_{D_0}^2} = C\sqrt{\varepsilon^{\frac{\alpha}{2}} (\|\omega_x\|^2 + \frac{\gamma}{\varepsilon^{\alpha}} \|\omega\|_{D_0}^2)} \le$$
$$\delta\left(\|\omega_x\|_{D_0} + + \frac{\gamma}{\varepsilon^{\alpha}} \|\omega\|_D^2\right) + \varepsilon^{\frac{\alpha}{2}} C_{\delta}, 0 < \alpha_0 < 1. \quad (13)$$

On the other side,

$$\|\omega_x\|_{D_0}^{\alpha_0}\|\omega\|_{D_0}^{1-\alpha_0} = C\|\omega_x\|_{D_0}^{\alpha_0}\|\omega_t\|_{L_2(0,T;D_0)}^{1-\alpha_0} \leq\leq \delta\left(\|\omega_x\|_{D_0}^2 + \frac{1-\gamma}{\varepsilon^\beta}\|\omega_t\|_{L_2(0,T;D_0)}^2\right) + \varepsilon^{\frac{\beta}{2}}C_\delta, \quad (14)$$

By joining (13) and (14), we obtain

$$\|\omega\|_{L_2(S)} \leq \delta\left(\|\omega_x\|_{D_0}^2 + \frac{\gamma}{\varepsilon^\alpha}\|\omega\|_{D_0}^2 + \frac{(1-\gamma)}{\varepsilon^\beta}\|\omega_t\|_{L_2(0,t;L_2(D_0))}^2\right) + C_\delta\varepsilon^{x}, \quad (15)$$

In the same way lets estimate the additives

$$\int_0^t \left|\int_S \frac{\partial v_t}{\partial n}\omega ds\right| dt \leq \int_0^t \left\|\frac{\partial v_t}{\partial n}\right\|_{L_2(S)} \|\omega\|_{L_2(S)} ds \leq \left(\int_0^t \left\|\frac{\partial v_t}{\partial n}\right\|_{L_2(S)}^2 dt\right)^{\frac{1}{2}}$$

$$\left(\int_0^t \|\omega\|_{L_2(S)}^2 ds\right)^{\frac{1}{2}} \leq C\left(\int_0^t \|\omega\|_{L_2(S)}^2 ds\right)^{\frac{1}{2}} \leq\leq \delta\int_0^t \left(\|\omega_x\|^2 + \frac{\gamma}{\varepsilon^\alpha}\|\omega\|^2 + \frac{(1-\gamma)}{\varepsilon^\beta}\|\omega_t\|^2\right) dt + C_\delta\varepsilon^{x}. \quad (16)$$

EVALUATION OF NONLINEAR ADDITIVES WITH THE FICTITIOUS DOMAIN METHOD

$$|(|v^\varepsilon|^p v^\varepsilon - |v|^p v, \omega_t)_D| = (\rho+1)|((v^\varepsilon\alpha_1 + v\beta_1)^\rho\omega, \omega_t)| \leq C(\|v^\varepsilon\|_{L_{\rho n(D)}} + +\|v\|_{L_{\rho n(D)}})\|\omega\|_{L_q(D)}\|\omega_t\|_{L_2(D)}. \quad (17)$$

Here

$$\alpha_1 + \beta_1 = 1, \alpha_1 \geq 0, \beta_1 \geq 0, \frac{1}{n} + \frac{1}{q} + \frac{1}{2} = 1$$

According to the conditions of the theorem left side (17) is estimated on $C\|\omega_x\|\|\omega_t\|_D$ and from (11)-(17) we take

$$\frac{1}{2}\frac{d}{dt}\left(\|\omega_x\|_D^2 + \|\omega_t\|_D^2 + \frac{\gamma}{\varepsilon^\alpha}\|\omega_t\|_{D_0}^2\right) + \frac{1-\gamma}{\varepsilon^\beta}\|\omega_t\|_{D_0}^2 \leq \delta\left(\|\omega_x\|^2 + \frac{\gamma}{\varepsilon^\alpha}\|\omega\|_{D_0}^2 + \frac{1-\gamma}{\varepsilon^\beta}\|\omega_t\|_{L_2(0,T;L_2(D_0))}^2\right) + C\varepsilon^{x}.$$

From this case the following estimation appears

$$\|\omega_x\|_{L_\infty(0,T;L_2(D))}^2 + \|\omega_t\|_{L_\infty(0,T;L_2(D))}^2 + \frac{\gamma}{\varepsilon^\alpha}\|\omega\|_{L_\infty(0,T;L_2(D_0))}^2 + \frac{(1-\gamma)}{\varepsilon^\beta}\|\omega_t\|_{L_2(0,T;L_2(D_0))}^2 \leq C\varepsilon^{x}. \quad (18)$$

$$\|\omega\|_{L_\infty(0,T;L_2(S))} \leq C\varepsilon^{x}.$$

By diffentiating (10) on t and taking $\varphi = \omega_{tt}$ with (4), we obtain the following estimation

$$\frac{1}{2}\frac{d}{dt}\left(\|\omega_{tt}\|_D^2 + \|\omega_{tx}\|_D^2 + \frac{\gamma}{\varepsilon^\alpha}\|\omega_t\|_{D_0}^2\right) + \frac{1-\gamma}{\varepsilon^\beta}\|\omega_{tt}\|_{D_0}^2 \leq \delta\left(\|\omega_{tx}\|_D^2 + + \frac{\gamma}{\varepsilon^\alpha}\|\omega_t\|_{D_0}^2\right) + \frac{1-\gamma}{\varepsilon^\beta}\|\omega_{tt}\|_{L_2(0,t;L_2(D_0))}^2 + C\varepsilon^{x} + |(\rho+1)(|v^\varepsilon|^p v_t^\varepsilon - |v|^p v_t, \omega_{tt}^\varepsilon)|,$$

In this case we consider that

$$\int_0^T \left\|\frac{\partial v_{tt}}{\partial n}\right\|_{L_2(S)}^2 dt$$

is limited.

From this equation the following estimation is formed

$$\|\omega_{tt}\|^2_{L_\infty(0,T;L_2(\Omega))} + \|\omega_{tx}\|^2_{L_\infty(0,T;L_2(D))} + \frac{\gamma}{\varepsilon^\alpha}\|\omega_t\|^2_{L_\infty(0,T;L_2(D_0))} + \frac{(1-\gamma)}{\varepsilon^\beta}\|\omega_{tt}\|^2_{L_2(0,T;L_2(D_0))} + \|\omega_t\|_{L_\infty(0,T;L_2(S))} \leq C\varepsilon^x. \quad (19)$$

The following theorem is obtained.

CONCLUSIONS

- In the development of the fictitious domain method can be divided into four interrelated areas:
- study different ways to continue the initial problems in a fictitious area;
- obtain best possible estimates in stronger metrics;
- extension of the class of problems for the application of the fictitious domain method;
- Building Effective difference schemes for the solution of the auxiliary problem of constructing a fictitious domain method.
- This paper covers the use of the fictitious domain method to complex non-linear problems, some aspects of numerical implementation for the auxiliary problem.

REFERENCES

1. Smagulov S., Sirochenko V. N., Orunkhanov M. K. (2001). *The numerical research of the liquid flow in regular areas,* Kazakh National University named after Al-Farabi, Inc. Almaty, Kazakhstan.
2. Konovalov A. N. (1972). The method of fictitious areas in filtration problems of two-phase liquid in accordance with the capillary forces, The numerical methods of mechanics, Vol. 3 No. 5.
3. Vabishevich P. N. (1991). *The method of fictitious areas in problems of mathematical physics,* Moscow State University, Inc. Moscow.
4. Lyons G. L.(1972). *The methods of non-linear limited problem solutions,* The world, Inc. Moscow.
5. Antoncev S. N., Kazhikov A.V., Monakhov V.N. (1975) *The bordered problems nonhomogeneous liquid mechanics,* NGU special courses, Inc. Novosibirsk.
6. Mikhailov V. P. (1976). *Differential equations of derivatives,* Science, Inc. Moscow.
7. He, J. H. (1997). Variational iteration method for delay differential equations, Commun. Nonlinear Sci. Numer. Simul, Vol. 2, pp. 235-236.